# Speaking of quantifiers

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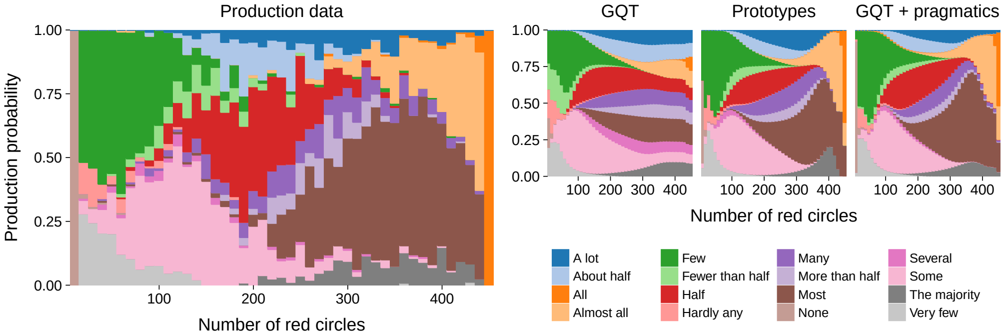
The theory of generalised quantifiers (GQT) is an influential theory about the meaning of quantity words, such as ‘some’, ‘most’, and ‘all’. According to GQT, quantity words express relations between sets. Thus, e.g., ‘Some circles are red’ is taken to mean that the set of circles shares at least one element with the set of red things, i.e., it is taken to mean that the *intersection set size* is greater than or equal to one.

Although GQT gives a compelling account of the *meaning* of quantity words, it seems less successful when it comes to describing the *use* of quantity words in natural language. To illustrate, we asked 600 participants on Mechanical Turk to each describe 10 displays showing 432 red and black circles. The proportion of red circles was varied across trials. To describe these displays, participants had to complete the sentence frame ‘— of the circles are red’. The left panel of the figure shows the binned production probabilities of the 16 most frequently produced quantity words.



A simple way of connecting GQT with the production of quantity words assumes that speakers describe quantities by randomly producing quantity words whose set-theoretic definitions are satisfied. However, there are at least three observations that speak against this idea: (i) the range of situations in which quantity words were produced was often much smaller than the range of situations in which their set-theoretic definitions were satisfied, see, e.g., ‘some’, (ii) quantity words were sometimes produced in situations in which their GQ-theoretic definitions were not satisfied, see, e.g., ‘more than half’, and (iii) the production probabilities for all quantity words formed gradients, which is at odds with the GQ-theoretic idea that quantified sentences are always true or false simpliciter.

In part, these observations may be explained by appealing to lexical and perceptual biases. At the lexical level, quantity words may vary in their *salience*, e.g., speakers may be more likely to consider ‘most’ than ‘more than half’. At the perceptual level, it has been shown that the Approximate Number System—the cognitive module used to estimate large numerosities—is not infallible (e.g., Dehaene, 1997). Hence, participants’ estimates of the number of red and black circles in the displays may have been inaccurate.

Combining these ingredients, we may define a simple GQ speaker $S_{GQ}$ as follows:

$$P_{S_{GQ}}(m \mid t; \theta, w) \propto \sum_{t' \in T} \overbrace{B_\theta(m, t')}^{\text{truth value}} \; \overbrace{P_{SL}(m)}^{\text{salience}} \; \overbrace{P_C(t' \mid t; w)}^{\text{ANS}}$$

The probability that $S_{GQ}$ produces a quantity word $m \in \{m_{\text{a lot}}, \ldots, m_{\text{very few}}\}$ given an intersection set size $t \in \{t_0, \ldots, t_{432}\}$ depends on three factors: the truth value of $m$ in $t'$ given $m$’s meaning $\theta_m$, the salience of $m$, and the probability of confusing the actual intersection set size $t$ for $t'$ given the accuracy of participants' estimates $w$. We simplifyingly assume that $\theta_m$ is a lower or upper bound on the intersection set size, depending on $m$'s *monotonicity*. Experiments were carried out to determine these monotonicity properties and to parametrise $w$, i.e., the accuracy of participants' estimates. Data-driven inference was used to determine the position of the bounds and the salience values.

The optimised production probabilities for $S_{GQ}$ are shown in the second panel of the figure. These predictions fail to account for much of the gradience in the production data, see, e.g., 'most'. The overall correlation between predictions and data was .704.

One might infer from the descriptive inadequacy of $S_{GQ}$ that GQT and its binary notion of truth should be rejected in favour of a *fuzzy*, i.e., graded notion of truth (e.g., Zadeh, 1983). Thus, it may be argued that quantified sentences are maximally true for a certain intersection set size, and that their truth value decreases gradually with the difference between this *prototype* and the actual intersection set size. Based on this fuzzy theory of quantification, we define a prototype speaker $S_{PT}$ who produces quantified sentences with a probability that is proportional to their fuzzy truth values. $S_{PT}$ associates each quantity word $m$ with two parameters: a prototype $p$ and a distance metric $\sigma$ modulating the effect of distance from the prototype on the truth value of $m$. Otherwise, $S_{PT}$ is identical to $S_{GQ}$.

The optimised production probabilities for $S_{PT}$ are shown in the third panel of the figure. These are indeed substantially more accurate than those of $S_{GQ}$ ($r = .933$), which may be taken as evidence that GQT should be replaced by a fuzzy theory of quantification to account for the production of quantity words. However, we argue that GQT in fact provides a superior account of the production of quantity words, but only if it is embedded in a probabilistic model of goal-oriented, i.e., pragmatic communication.

Whereas $S_{GQ}$ randomly produces truthful quantity words, its probabilistic pragmatic counterpart $S_{GQ+}$ has a preference for producing more *informative* messages. To operationalise informativity, we first define a simple hearer $H_{GQ}$, who, upon hearing a quantity word, randomly infers an intersection set size that is compatible with its set-theoretic meaning: $P_{H_{GQ}}(t \mid m) \propto B_\theta(m,t)$. $S_{GQ+}$ prefers to produce quantity words that increase the probability that $H_{GQ}$ infers the correct intersection set size (Frank & Goodman, 2012). Thus, e.g., if all of the circles are red, $S_{GQ+}$ prefers producing 'all' over 'most'.

In this way, $S_{GQ+}$ may be defined as follows. Higher values of the free parameter $\lambda$ capture that the speaker is more likely to optimise the probability of coordination.

$$P_{S_{GQ+}}(m \mid t; \theta, w, \lambda) \propto \sum_{t' \in T} \overbrace{P_{H_{GQ}}(m \mid t'; \theta)^{\lambda}}^{\text{hearer behaviour}} \; \overbrace{P_{SL}(m)}^{\text{salience}} \; \overbrace{P_C(t' \mid t; w)}^{\text{ANS}}$$

The optimised production probabilities for $S_{GQ+}$ are shown in the right panel of the figure. These provide an excellent fit to the data ($r = .958$), surpassing even those of $S_{PT}$. Moreover, $S_{GQ+}$ is more parsimonious than $S_{PT}$ since it associates quantity words with one rather than two meaning parameters.

Thus, we have formulated an integrated pragmatic-cognitive computational model of the production of quantity words based on the GQ-theoretic assumption that quantity words express relations between sets, and have shown that this model provides a compelling account of the production of quantity words in English.

**References:** **[1]** Barwise, J. & Cooper, R. (1981) *L&P, 4*. **[2]** Dehaene, S. (1997). *The number sense*. **[3]** Frank, M. & Goodman, N. (2012). *Science*, 336. **[4]** Zadeh, L. (1983). *Comput Math Appl, 9*.